Today’s Plan

• Review of Last Week: Midterm
• Basic IV Analysis
• IV Implementation: 2SLS
Basic IV Analysis
Motivation Question

- Evaluating the causal impact of charter schools
  - If students in a charter schools forms a treatment group, what makes a good control group?
- Naive comparison–compare students from a charter school with students from nearby public schools–reveal positive impact on test scores.
  - Is this causal?
  - How to reach a causal conclusion?
Charter Lottery

- While we usually can’t run certain experiments, there’re experiments generated by public policies
- MA Law: Scare charter seats should be allocated by lottery
- With a random lottery, we can now compare applicants who are offered a seat with applicants who are not offered a seat
  - But what if we’re interested in the causal effect of attendance on test scores?
- Let’s first see what we can get from data
## Lottery Table

<table>
<thead>
<tr>
<th></th>
<th>Lynn public fifth graders (1)</th>
<th>KIPP Lynn lottery winners (2)</th>
<th>Winners vs. losers (3)</th>
<th>Attended KIPP vs. others (4)</th>
<th>Attended KIPP vs. others (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A. Baseline characteristics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hispanic</td>
<td>.418</td>
<td>.510</td>
<td>-.058</td>
<td>.539</td>
<td>.012</td>
</tr>
<tr>
<td>Black</td>
<td>.173</td>
<td>.257</td>
<td>.026</td>
<td>.240</td>
<td>-.001</td>
</tr>
<tr>
<td>Female</td>
<td>.480</td>
<td>.494</td>
<td>-.008</td>
<td>.495</td>
<td>-.009</td>
</tr>
<tr>
<td>Free/Reduced price lunch</td>
<td>.770</td>
<td>.814</td>
<td>-.032</td>
<td>.828</td>
<td>.011</td>
</tr>
<tr>
<td>Baseline math score</td>
<td>-.307</td>
<td>-.290</td>
<td>.102</td>
<td>-.289</td>
<td>.069</td>
</tr>
<tr>
<td>Baseline verbal score</td>
<td>-.356</td>
<td>-.386</td>
<td>.063</td>
<td>-.368</td>
<td>.088</td>
</tr>
<tr>
<td><strong>Panel B. Outcomes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Attended KIPP</td>
<td>.000</td>
<td>.707</td>
<td>.741</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Math score</td>
<td>-.363</td>
<td>-.000</td>
<td>.355</td>
<td>.095</td>
<td>.467</td>
</tr>
<tr>
<td>Verbal score</td>
<td>-.417</td>
<td>-.262</td>
<td>.113</td>
<td>-.211</td>
<td>.211</td>
</tr>
<tr>
<td>Sample size</td>
<td>3,964</td>
<td>253</td>
<td>371</td>
<td>204</td>
<td>371</td>
</tr>
</tbody>
</table>
IV Assumptions

• With a few assumptions, the lottery can give us the causal effect of attendance on test scores
• IV-AS.1 [First Stage]: The instrument has a causal effect on the variable whose effects we’re trying to capture, in this case KIPP enrollment.
• IV-AS.2 [Independence] The instrument is randomly assigned or “as good as randomly assigned”.
• IV-AS.3 [Exclusion Restriction] The exclusion restriction describes a single channel through which the instrument affects outcomes.
• These assumptions leads to the following IV chain
IV Chain Reaction

Effect of offers on scores =

• \{Effect of offers on attendance\} \times \{Effect of attendance on scores\}

• \implies Effect of attendance on scores = ?

• Based on table 3.1, we have the effect of KIPP attendance on math scores:

\[
\frac{0.355}{0.741} = 0.486
\]
Test

• Based on Table 3.1, derive the effect of KIPP attendance on verbal scores
Terms

1. The original randomizer (a KIPP offer): an instrumental variable
2. The first link: First stage (1st)
3. The whole link: Reduced form (RF)
4. The second link: Local Average Treatment Effect (LATE)
Data

• To run an IV analysis, we usually need the following variables:
  1. treatment variable: \( D_i \)
  2. outcome variable: \( Y_i \)
  3. instrumental variable(s): \( Z_i \)

• IV chain: \( Z_i \rightarrow Y_i = \{Z_i \rightarrow D_i\} \times \{D_i \rightarrow Y_i\} \)
Formal Representation

• 1st:
  \[ \phi = E[D_i \mid Z_i = 1] - E[D_i \mid Z_i = 0] \]
  denotes difference in KIPP attendance rate between those who were and were not offered a seat in the lottery

• RF:
  \[ \rho = E[Y_i \mid Z_i = 1] - E[Y_i \mid Z_i = 0] \]
  denotes difference in test scores between those who were and were not offered a seat in the lottery

• LATE
  \[ \lambda = \frac{\rho}{\phi} \]
Types of People

- In order to understand the “local” part of the LATE, it’s necessary to consider 4 groups of people:
  1. Never takers: Subpopulation with $D_i = 0$ regardless of value of $Z_i$
  2. Always takers: Subpopulation with $D_i = 1$ regardless of value of $Z_i$
  3. Compliers: Subpopulation with $D_i = 0$ if $Z_i = 0$ and $D_i = 1$ if $Z_i = 1$
  4. Defiers: Subpopulation with $D_i = 1$ if $Z_i = 0$ and $D_i = 0$ if $Z_i = 1$

- In practice, the group of defiers is hard to imagine, a final assumption of an IV design is
  - IV-AS.4: No defiers
Test:

- 1st
  \[ \phi = E[D_i \mid Z_i = 1] - E[D_i \mid Z_i = 0] \]
- What’s the 1st for the four groups of people?
  - Never takers
  - Always takers
  - Compliers
  - Defiers
LATE

- The 1st is only driven by compliers, so LATE reflects average causal effect in this group
- Let $C_i = 1$ denotes the group of compliers, our new causal parameter is

$$\text{LATE} = E[Y_{1i} - Y_{0i} \mid C_i = 1] = \lambda = \frac{\rho}{\phi}$$

- Note that LATE tells us nothing about never takers and always takers
- Is the population of compliers an interesting group of people?
- What's the relationship between LATE and ATET or ATEN?
Summary

• What is an IV analysis?
  ■ IV = reduced-form comparisons across groups defined by the instrument, scaled by the first stage.

• What are the ID assumptions?
  1. Existence of 1st
  2. The instrument is as good as randomly assigned
  3. Exclusion restriction
  4. No defiers
IV Implementation: 2SLS
Motivation Question

- As an example to understand two-stages least squares, let’s consider the following questions:
  - Is there a causal relationship between family size and living standard?
  - How do we quantify this relationship credibly?
- In a sense, answers to this question provide insights for population policies in many developing countries
- Also gives a test to Gary Becker’s “quantity-quality fertility model”
Naive Comparisons

• Questions:
  ▪ What kinds of comparisons would you make?
  ▪ Treatment variable?
  ▪ Outcome variable?

• Suppose I just compare, say, labor supply outcomes from women with larger families with those with smaller families, what’re the problems? What about adult firstborns’ highest grade?
Casual Regressions

- Angrist, Lavy, and Schlosser (2010) run a regression of adult firstborns’ highest grade completed on family size, controlling for age and sex:
  - An extra sibling is associated with a reduction of about one-quarter of a year of schooling
- What’re the possible omitted variables?
Ideal Experiment

- Source of selection bias: family size is endogenous
- To get rid of all the selection biases, one might run the following ideal experiment:
  1. Draw a sample of families with one child
  2. In some of these households, randomly distribute an additional child
  3. Wait 20 years and collect data on the educational attainment of firstborns who did and did not get an extra sibling
- Can we really run this ideal experiment?
The Twins Experiment

- Consider a group of first-born adults in a random sample of men and women born to mothers with at least two children
- Control group: First-born adults from families in which the second born is a singleton
- Treatment group: First-born adults from families in which the second borns are twins
1st and RF

• 1st:
  ▪ On average, families in which the second born is a singleton include 3.6 children
  ▪ On average, families in which the second borns are twins include 3.92 children

• RF:
  ▪ Highest grades of adult firstborns from the first kind of families
  ▪ Highest grades of adult firstborns from the second kind of families
The Same-Sex Experiment

• In many countries, fertility is affected by sibling sex composition:
  ▪ Families whose first two children are both boys or both girls are more likely to have a third child
• What’s the instrument here?
• 1st?
• RF?
Potential Failures

• Threat to the first research design:
  ▪ Twins are not that random: Multiple births are more frequent among mothers who are older and for women in some racial and ethnic groups
  ▪ But perhaps we can add control variables such as maternal age

• Threat to the second research design:
  ▪ Perhaps the sex-mix of the first two children affects children’s educational outcomes for other reasons?
ER Testing

- The ER is not easily verified, but it has an interesting implication for us to exploit:
  - In a sample where the 1st is small, we would expect the RF will also be small if exclusion restriction holds
  - Always-takers and never-takers have a zero 1st
- Example:
  - Religious women might always plan to have three or more children
  - Highly educated women might always plan to have small families
IV Extension: ILS and 2SLS

• Just as regression generalizes naive mean comparison, 2SLS also generalizes basic IV analysis:
  ▪ Multiple instruments
  ▪ Control for covariates

• In practice we will run 2SLS, but to refresh our regression knowledge, let’s also look at ILS, which also an intuitive way to obtain IV results
ILS is IV: RF

• Recall that RF is
  \[ \rho = \mathbb{E}[Y_i \mid z_i = 1] - \mathbb{E}[Y_i \mid z_i = 1], \]
  but you can also run a RF regression to get \( \rho \)
  \[ Y_i = \alpha_0 + \rho z_i + e_{0i}. \]
ILS is IV: 1st

- Recall that 1st is
  \[ \phi = E[D_i \mid z_i = 1] - E[D_i \mid z_i = 1], \]
  but you can also run a 1st regression to get \( \phi \)
  \[ D_i = \alpha_1 + \phi z_i + e_{1i}. \]
2SLS is IV: 1st

- 2SLS also runs a 1st regression
  \[ D_i = \alpha_1 + \phi Z_i + e_{1i}, \]
  but the goal is to get the 1st fits:
  \[ \hat{D}_i = \alpha_1 + \phi Z_i. \]
2SLS is IV: 2nd

- The 1st fits are used as a regressor for the second stage (2nd) regression:
  \[ Y_i = \alpha_2 + \lambda_{2SLS} \hat{D}_i + e_{2i} \]
- The value of \( \lambda_{2SLS} \) is identical to the ratio of RF to 1st regression coefficients, \( \frac{\rho}{\phi} \):
  - Derivation
2SLS: Adding Controls

- Suppose we add maternal age as an control variable for our twin instrument:
  - RF: $Y_i = \alpha_0 + \rho Z_i + \gamma_0 A_i + e_{0i}$
  - 1st: $D_i = \alpha_1 + \phi Z_i + \gamma_1 A_i + e_{1i}$
  - 1st fits: $\hat{D}_i = \alpha_1 + \phi Z_i + \gamma_1 A_i$
  - 2nd: $Y_i = \alpha_2 + \lambda_{2SLS} \hat{D}_i + \gamma_2 A_i + e_{2i}$
2SLS: Multiple Instruments

- We can also include the two instruments to increase efficiency:
  - RF: \( Y_i = \alpha_0 + \rho_t Z_i + \rho_s W_i + \gamma_0 A_i + \delta_0 B_i + e_{0i} \)
  - 1st: \( D_i = \alpha_1 + \phi_t Z_i + \phi_s W_i + \gamma_1 A_i + \delta_1 B_i + e_{1i} \)
  - 1st fits: \( \hat{D}_i = \alpha_1 + \phi_t Z_i + \phi_s W_i + \gamma_1 A_i + \delta_1 B_i \)
  - 2nd: \( Y_i = \alpha_2 + \lambda_{2SLS} \hat{D}_i + \gamma_2 A_i + \delta_2 B_i + e_{2i} \)

- \( \lambda_{2SLS} \) is a weighted average of the estimates we’d get using \( Z_i \) and \( W_i \) one at a time
Understanding Reg Tab

<table>
<thead>
<tr>
<th></th>
<th>Twins instruments</th>
<th>Same-sex instruments</th>
<th>Twins and same-sex instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Second-born twins</td>
<td>.320</td>
<td>.437</td>
<td>.449</td>
</tr>
<tr>
<td></td>
<td>(.052)</td>
<td>(.050)</td>
<td>(.050)</td>
</tr>
<tr>
<td>Same-sex sibships</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.079</td>
<td>.073</td>
<td>.076</td>
</tr>
<tr>
<td></td>
<td>(.012)</td>
<td>(.010)</td>
<td>(.010)</td>
</tr>
<tr>
<td>Male</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-.018</td>
<td>-.020</td>
<td>-.020</td>
</tr>
<tr>
<td></td>
<td>(.010)</td>
<td>(.010)</td>
<td>(.010)</td>
</tr>
<tr>
<td>Controls</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td></td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 3.5
OLS and 2SLS estimates of the quantity-quality trade-off

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>OLS estimates (1)</th>
<th>Twins instruments (2)</th>
<th>Same-sex instruments (3)</th>
<th>Twins and same-sex instruments (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Years of schooling</td>
<td>-.145 (.005)</td>
<td>.174 (.166)</td>
<td>.318 (.210)</td>
<td>.237 (.128)</td>
</tr>
<tr>
<td>High school graduate</td>
<td>-.029 (.001)</td>
<td>.030 (.028)</td>
<td>.001 (.033)</td>
<td>.017 (.021)</td>
</tr>
<tr>
<td>Some college (for age ≥ 24)</td>
<td>-.023 (.001)</td>
<td>.017 (.052)</td>
<td>.078 (.054)</td>
<td>.048 (.037)</td>
</tr>
<tr>
<td>College graduate (for age ≥ 24)</td>
<td>-.015 (.001)</td>
<td>-.021 (.045)</td>
<td>.125 (.053)</td>
<td>.052 (.032)</td>
</tr>
</tbody>
</table>
IV Demo
Load Data

```
rio::import("http://econ300.com/fertility.dta") -> fertility
```
Basic 2SLS

\[
\text{AER::ivreg(weeksml ~ morekids | samesex, data = fertility)}
\]

Call:
AER::ivreg(formula = weeksml ~ morekids | samesex, data = fertility)

Coefficients:
(Intercept) morekids
 21.42  -6.31
2SLS with Covariates

AER::ivreg(weeksml ~ morekids + ageml | samesex + ageml, data = fertility)

Call:
AER::ivreg(formula = weeksml ~ morekids + ageml | samesex + ageml, data = fertility)

Coefficients:
(Intercept) morekids      ageml
  -3.126   -6.061    0.804
References